Asymptotic Modelling of Metal Forming

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1 Introduction

Control systems are able to monitor manufacturing processes to inexpensively vary each product and ensure high quality, despite material imperfections and machine wear. Such control systems depend on an understanding of the ongoing process in the form of a predictive model that can be computed in real time or faster. Current design processes using finite element models are computationally too slow, giving rise to the need for faster analytical models. Collaborators in CUED are researching practical control of ring rolling and spinning, but as a means of understanding the key issues, and in order to build on existing literature, an investigation into flat sheet rolling is being undertaken. This will comprise of a 'Demonstrator' rolling and bending machine, mathematical modelling and simulation. This has been the focus of my enquiry thus far.

2 Background

Modelling of rolling began in early twentieth centuary Germany with noteable publications including Siebel (1924), Siebel and Pomp (1927), Karman (1925) and Nadai (1939). The next major contribution came from Orowan (1943) in which an approximate model is presented that incorporates shear. This solution has been generally adopted as a benchmark and is widely used by industry, although with empirically fitted parameters. Some developments since then include Orowan and Pascoe (1946), Bland and Ford (1948) and Bland et al. (1948); and some work based on slip-line theory such as Alexander et al. (1955) and Collins (1969). Hartley et al. (1989) provides the first review of rolling which predominantly covers this classical modelling as well as experimentation and early finite element simultion.

Many numerical studies have been performed since they began in 1972 with Alexander (1972). Finite element simulations have since become very popular and several reviews exist including Montmitonnet and Buessler (1991), Domanti and McElwain (1998) and Montmitonnet (2006). It is, in fact, included as an example problem in the '*ABAQUS* Example Problems Manual' (Dassault Systemes, 2012).

2.1 Asymptotic Methods

More recently, asymptotic methods have been applied to solve a complete set of governing equations in the limit of a small aspect ratio: thin sheets and large rollers. It was first utilised in metal forming, specifically conical extrusion, in 1987 by Johnson. Johnson is a named author on numerous publications that apply the same techniques to sheet rolling. The first of these is Smet and Johnson (1989), followed three years later by Johnson and Smelser (1992). The former applies an almost identical process to that in Johnson (1987) while also neglecting elasticity. The major distinctions from extrusion being in the geometry and direction of friction: as the rollers rotate there is a neutral point at which the velocity of the material and rollers match, indicating a change in direction of friction. The latter makes a number of simplifications to the former to progress further to a closed form solution; a rigid-plastic material with yield stress as an arbitrary function is used and the magnitude of surface friction is set to be a constant fraction of the yield stress.

Although yielding single integral solutions, these assumptions may fail to accurately capture the physics of the problem, especially the friction model. This motivated a similar formulation in Domanti and McElwain (1995), who re-introduces the more commonly used Coulomb friction, while assuming the ratio of maximum pressure to yield stress is large and the reduction is small. Finally, the most comprehensive two-dimensional formulation to date is presented in Cherukuri et al. (1997). Using a relative-slip friction model and strain-rate dependent constitutive equations, the governing equations are solved to ODEs, assuming only a small aspect ratio. This is repeated for small, medium and large friction and no-slip conditions.

Asymptotic approaches to three dimensional effects and spread are Johnson (1991) and Domanti et al. (1994). Asymptotic analysis has also been used for stability analysis of 'chatter' in Johnson (1994); a multiple scales analysis of work roll heat transfer in Johnson and Keanini (1998); and a model for roller deformation in Langlands and McElwain (2002). Other considerations include surface finish, heating and residual stresses. Reviews are presented in Montmitonnet and Buessler (1991), Montmitonnet (2006) and Domanti and McElwain (1998).

3 Asymptotic Modelling

All the asymptotic solutions begin with a set of governing equations describing force balance, mass conservation and constitutive laws. Elasticity is neglected so the relevant area to solve is solely within the roll bite, shaded in fig. 1, and it is assumed that this entire region has reached the yield condition.

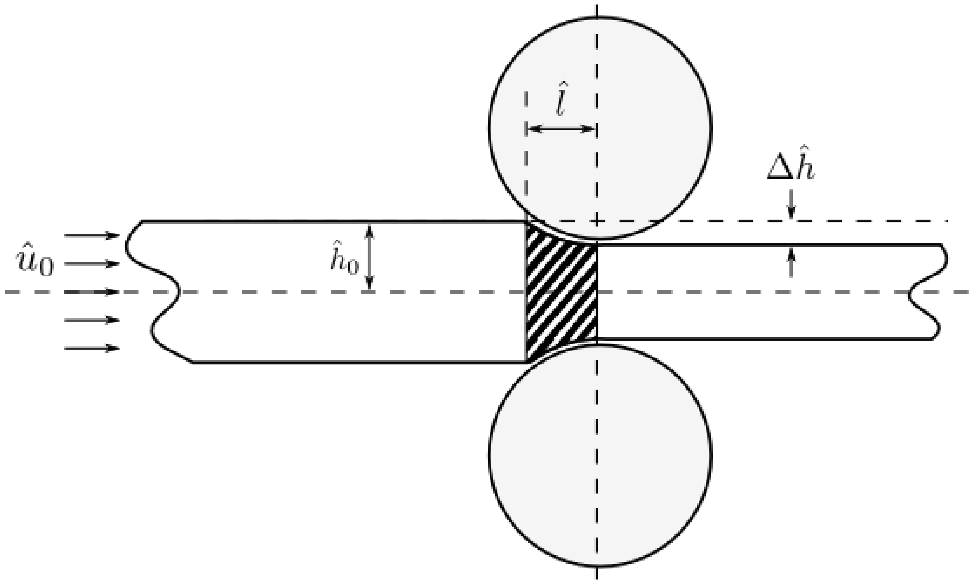

Figure 1: A not to scale diagram of a symmetric roller set-up, highlighting the plastic region to be solved (shaded) and other key parameters.

The governing equations for rigid plastic are then

$$-\frac{\partial\hat{p}}{\partial\hat{x}} + \frac{\partial\hat{S}_{xx}}{\partial\hat{x}} + \frac{\partial\hat{S}_{xy}}{\partial\hat{y}} = 0,$$

$$-\frac{\partial\hat{p}}{\partial\hat{y}} + \frac{\partial\hat{S}_{yy}}{\partial\hat{y}} + \frac{\partial\hat{S}_{xy}}{\partial\hat{x}} = 0,$$

$$\frac{\partial\hat{v}_x}{\partial\hat{x}} + \frac{\partial\hat{v}_y}{\partial\hat{y}} = 0,$$

$$\frac{\partial\hat{v}_x}{\partial\hat{x}} = \hat{\lambda}\hat{S}_{xx},$$

$$\frac{\partial\hat{v}_y}{\partial\hat{y}} = \hat{\lambda}\hat{S}_{yy},$$

$$\frac{\partial\hat{v}_x}{\partial\hat{y}} + \frac{\partial\hat{v}_y}{\partial\hat{x}} = 2\hat{\lambda}\hat{S}_{xy}, \text{ and}$$

$$\hat{S}_{xx}^2 + \hat{S}_{yy}^2 + 2\hat{S}_{xy}^2 = 2\hat{k}^2,$$

where p is pressure, S is the deviatoric stress, v is velocity, k is the yield stress in shear and λ is the flow rate. Hats denote dimensional quantities.

The first assumptions and scaling I considered was that of Domanti and McElwain (1995),

$$\begin{array}{ccc} \hat{x} = \hat{l}x & \hat{y} = \hat{h}_0 y & \hat{p} = \hat{p}_{\max} p \\ \hat{S}_{xx} = \hat{k} S_{xx} & \hat{S}_{yy} = \hat{k} S_{yy} & \hat{S}_{xy} = \mu \hat{p}_{\max} S_{xy} \\ \hat{u}_x = \hat{u}_0 u_x & \hat{u}_y = \hat{u}_0 u_y & \hat{\lambda} = \frac{r u_0}{\hat{k}\hat{l}} \lambda, \end{array}$$

where $\delta = \hat{h}/\hat{l} \ll 1$, $r = \Delta\hat{h}/\hat{h}_0 = O(\delta)$, $\mu = O(\delta)$ and $\hat{k}/\hat{p}_{\max} = O(\delta)$. It combines the simplest material and friction models: rigid plastic and a small reduction coulomb friction. I generalised this model to weakly asymmetric roller sizes and friction co-efficients; weakly asymmetric meaning the ratio between roller sizes and frictions is of order one. To second order correction, this produced solutions similar to those in Domanti and McElwain (1995) except the friction and reduction terms become averaged in some sense between the rollers. For example,

$$p^{(0)} = p_0 e^{\frac{\mu}{\eta} \frac{1+\gamma}{2} x}$$

and

$$v_x{}^{(1)} = \frac{r^2}{\eta^2}(2x - x^2)\frac{1+\kappa}{2}$$

where γ is the ratio of roller frictions making $\frac{1+\gamma}{2}$ an average of both frictions. κ, the ratio of roller size, produces similar terms. Like Domanti and McElwain (1995), this could be solved to integral equations as far as the second order correction despite the algebra becoming unwieldy.

I also investigated modified boundary conditions to account for the effects of vertical forces and moments in addition to the previously considered horizontal forces. This found only limited success as vertical forces and moments are not supported at leading order. This is unsurprising as pressure dominates over the yield stress and so leading order vertical forces or moments would result in a stretching or bending of the sheet - independently of the rolling process. It was also observed that zero or positive tension resulted in no or negative pressure peaks. This is due to the feedback between surface friction and pressure, so without a sufficiently high boundary pressure, from compression end conditions, the pressure does not climb to an order of magnitude larger than the longitudinal deviatoric stresses. This is possibly why the neutral point and maximum pressures are fitted empirically in Domanti and McElwain (1995) and is a major limitation of this model.

As a potential fix, I rescaled the problem such that pressure was of order $\hat{k}$. This was solved in a similar manner and resulted in the pressure and longitudinal deviatoric stresses balancing at leading order, allowing tension conditions as high as the yield condition.

Both these solutions can be recast to resemble two of the four solution regimes of Cherukuri et al. (1997): Domanti corresponds to medium friction and the rescaled model to low friction. Of course, Cherukuri et al. avoids the low pressure exponential trap at medium friction by using the relative slip friction model.

Using Cherukuri et al. (1997) as a frame work, I developed models for rigid-plastic without the small reduction assumption with both Coulomb and relative-slip friction models.

Table 1: Map of Modelling Assumptions

	Coulomb Friction	Relative Slip	No-Slip
Rigid-Plastic	$O(\delta)$	$O(\delta)$ and $O(1)$	
Rate Hardening		Cherukuri et al. (1997)	Cherukuri et al. (1997)
Work Hardening	Domanti et al. (1993)		

The scalings that differ from those above are

$$\hat{p} = \frac{\beta}{\delta}\hat{k}p \quad \hat{S}_{xy} = \beta\hat{k}S_{xy} \quad \hat{\lambda} = \frac{u_0}{\hat{k}\hat{l}}\lambda,$$

where $\beta = \hat{\tau}_0/\hat{S}_0$, $\hat{\tau}_0$ is a characteristic shear, and $\hat{S}_0 = \hat{k}$ is a characteristic longitudinal deviatoric stress.

This was successful for the low friction models. Solutions were found to a first order correction in terms of a single integral such as, for the relative-slip case,

$$p^{(0)} = \int \frac{U}{\Delta U}\frac{1}{h(x)} - \frac{1}{\Delta U}\frac{1}{h(x)^2}dx,$$

where U is the roller surface velocity and ΔU is the difference in charactistic velocity and roller velocity. Otherwise solutions were found in terms of an ODE such as, for the Coulomb friction case,

$$\frac{dp^{(0)}}{dx} = \frac{1}{h(x)}\mu p^{(0)}$$

which can be solved for $p^{(0)} = p_0 e^{\mu \int \frac{1}{h(x)} dx}$ in this instance but not generally for higher orders.

These also perform well, matching closely with finite element solutions, and indicate little variation between friction models, shown in fig. 2.

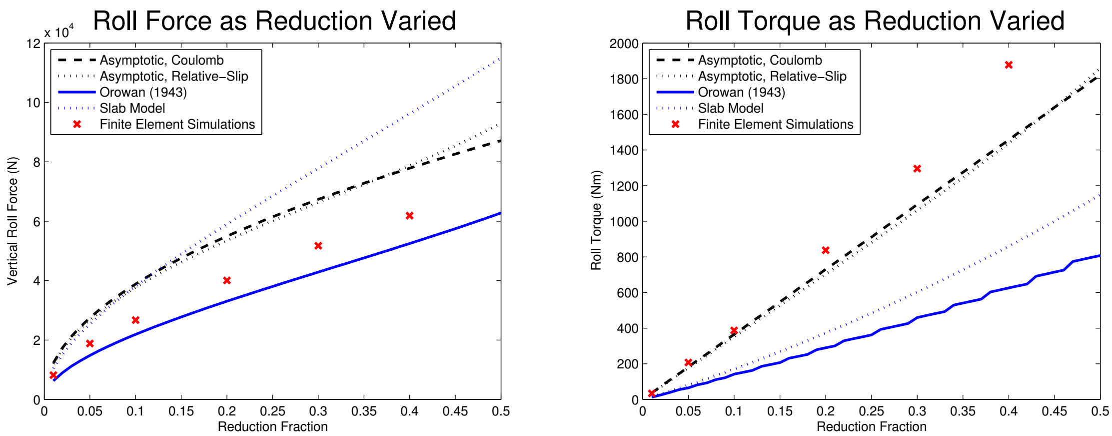

Figure 2: Results from a poster presentation at SNSCC-2014 showing a comparison of analytical models with finite element simulation results. Vertical roll force (left) and roll torque (right) as the workpiece reduction is varied.

Higher friction models continue to break under two conditions. Firstly; non-compressive boundary conditions discussed previously which may be fixed by introducing elasticity to facilitate a boundary layer to match the high pressure to zero pressure end conditions. Secondly; an intermediate range of friction which is sufficient for sticking on only part of the roller. Orowan (1943) performs a check at each iterative step to determine where sticking begins and Cherukuri et al. (1997) solves for the complete non-sticking and complete sticking cases but does not address this transition.

4 Future Work

There are many lines of enquiry for the study of rolling. After the construction of the 'Demonstrator' machine, experimental comparison would be possible and is work that is less common in exisitng literautre. Developing models to complete the first two rows of table 1 would also provide a valuable comparison of material and friction models if experimental results could be included.

Many opportunities also exist to incorporate more complex physics and phenomena into the models. One important course of investigation will be introducing elasticity; this should allow non-compressive and bending boundary conditions to be matched to the higher order friction solutions. This also introduces the possibility of determining residual stresses in the product. Roller deformation is also a major concern for cold rolling as this typically involves thinner sheets at much higher pressures. Of course, temperature is more significant in hot rolling but can also be affected by the dissipation of deformation energy. Early rolling of the product, usually hot, will also be of a thicker sheet, exceeding the validity of the asymptotic regime considered so far.

Numerous other processes could also benefit from investigation. Ring rolling control is currently being reasearched experimentally within the CUED and further work into the asymmetric rolling and bending boundary conditions could be applicable there. The major challenge in ring rolling would be the ring thickness as these are generally outside the asymptotic regime considered so far.

Another exciting modelling application is the English Wheel. It is an application that exists closer to the small aspect ratio regime so with the introduction of three dimensional effects, particularly around the roller ends, a solution similar to those presented here could be matched to an outer shell solution. That may be sufficient to generate a workpath for process automation for which nothing currently exists in literature.

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