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## Meta-Analysis of Curvature Trends in Asymmetric Rolling

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### Abstract

This paper investigates curvature in asymmetric rolling by combining existing results from eight experimental and nine numerical previous studies. These previous results are digitised and a linear regression model fitted which explains 65% of the variance in these data. It is found that conclusions from several previous studies are contradicted by other previous studies, and that there is no consensus on the fundamental mechanism of curvature generation in rolling. Results from an existing curvature-predicting analytic slab model are also compared with the previous results, and the agreement is shown to be adequate at best. Future work is clearly needed to enable accurate curvature prediction, and it is hoped that the evidence collected here will inform future investigations and models to ensure the relevant range of parameters are considered.

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*Keywords:* Asymmetric Rolling; Curvature Prediction; Meta-analysis

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### 1. Introduction

Workpiece curvature in rolling processes has significant impact on process design and product quality. Unintentional asymmetries cause turn-up and turn-down in at-sheet rolling which can halt the process or even damage the rolling table and mills; asymmetries are exploited in asymmetrical rolling to improve process efficiency and produce curved products; and uniform curvature must be maintained in ring-rolling to prevent elliptical rings forming and other instabilities. Consequently, experimental investigations into this area date back to at least 1956 [1]

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## Nomenclature

$\kappa$	workpiece curvature
$\delta$	roll gap aspect ratio
$r$	reduction
$U_t/U_b$	top and bottom roll surface speed ratio
$R_t/R_b$	top and bottom roll radius ratio
$\mu_t/\mu_b$	top and bottom workpiece-roll friction ratio
$\sigma_y$	yield stress in shear
$E$	Young's modulus
$A_{1-11}$	linear regression coefficients

and modelling of this phenomena to 1963 [2]. Numerical studies began much later, around 1988 [3], but have since proliferated owing to the comparatively low cost of simulations. Simulations have also provided higher levels of parameter precision, for example being able to specify friction coefficients, as well as providing complete stress and strain fields throughout the workpiece.

Nine [1, 4-11] experimental and [3, 12-20] numerical studies have been reviewed. Many of the conclusions found in these experimental studies are corroborated in the numerical studies, although several contradictions are also found. Some conclusions from the reviewed publications include:

- Curvature may be towards the faster roll [1, 4] or towards the slower roll [5, 9, 3]. The direction of curvature depends on the roll gap aspect ratio [14] and the reduction [6, 11].
- Curvature may be towards the smaller roll [7] or towards the larger roll [18]. The magnitude of curvature depends on the roll gap aspect ratio and the reduction [15].
- If the angular velocity ratio between the two rolls is held constant and the roll size (or equivalently the surface speed) ratio is varied, the direction of curvature depends on the roll gap aspect ratio and the reduction [13, 20]; thick-sheets curve towards the faster/larger rolls and thin sheets towards the slower/smaller rolls [16]. Past some limit, larger asymmetry will not produce more curvature [20].
- Curvature may be towards the roll with higher friction [12, 14], although the direction of curvature is dependent on the reduction [10].
- Curvature is found to be dependent on the material properties [15]; specifically, a smaller Young's modulus induces greater curvature [7].

Clearly a complex relationship exists between the roll parameters and curvature, for which no one individual study provides a clear picture. Some of the variation between studies may be due to unintended consequences of the different configurations investigated. For example, insufficiently wide workpieces can lead to edge effects dominating the results [5, 6, 8], which may also occur in 3d simulations [19]. Insufficiently long workpieces or too short simulations can result in end effects dominating the results [4, 8, 3, 12]. Variations in friction, including slipping and sticking transitions along the roll-workpiece interface, can radically influence curvature [8]. It is also plausible that other effects such as gravity or numerical instabilities may impact individual studies.

As can be seen, despite the long history, few generally applicable rules have been found describing the curvature produced by a given rolling mill configuration. Further, even in the most comprehensive investigations, asymmetry is typically introduced by varying only one parameter whilst the others are held symmetric and therefore possible effects of combinations of asymmetries are missed. Once multiple asymmetries are introduced concurrently the trends become unclear and conclusions from these limited data sets become questionable. This is clearly demonstrated from the loose conclusions drawn in the larger studies such as Johnson and Needham [6].

This study aims to present a more holistic view of curvature generation. Section 3 provides a quantitative review and comparison of experimental and numerical works, including the application of a linear regression, Section 4. A comparison with existing curvature predicting models within the literature is then given in Section 5.

## 2. Quantitative Comparison

The eight experimental studies and nine numerical studies were digitised to provide a comparison between studies. To be included, papers had to specify sufficient detail to determine the roll gap aspect ratio, reduction, friction ratio, roll speed ratio, roll size ratio, and material name or yield stress curve and Young's modulus.

The data is presented in Figure 1, grouped by publication and selected for the asymmetry varied in each study. Curvature is measured using a variety of different metrics in the various studies, some of which are specific to the geometry used [11]. Here, curvature is non-dimensionalised by workpiece thickness. Asymmetry is specified by taking the log of the ratio of various parameters; this gives the correct symmetry under vertical reflection of the set-up, and gives the value zero in the symmetric case.

Hardening was incorporated through an effective yield stress, taken as a mid-point on the yield stress curve or within ranges presented in third party data sheets. The workpiece-roll surface friction was typically characterised as the roll roughness in the experimental studies. It should be noted that, since there is no systematic way of determining an effective friction coefficient from the roll roughness alone, friction ratios for the experimental results were chosen to be representative values reasonable within the range used by the numerical studies.

Figure 1 shows some negative correlation between curvature and the roll speed ratio, and perhaps between curvature and the friction ratio. Upon further investigation more correlation can be described by also considering the reduction and roll gap aspect ratio. Definitive structure also exists for the roll radius/speed asymmetry and possibly the roll radius ratio. This structure was found to be dependent on the reduction. A nonlinear relation between the curvature and roll radius, roll speed and roll radius/speed asymmetries is also evident.

Unfortunately, many of these trends only present in individual papers resulting in data clustering and unclear connections between these studies. Combined with the dependence on numerous parameters unable to be displayed in a given plot, it is difficult to ascertain strong, generally applicable conclusions.

To more rigorously determine consistency between studies and account for the simultaneous variation of multiple parameters, a statistical regression analysis is performed in the next section.

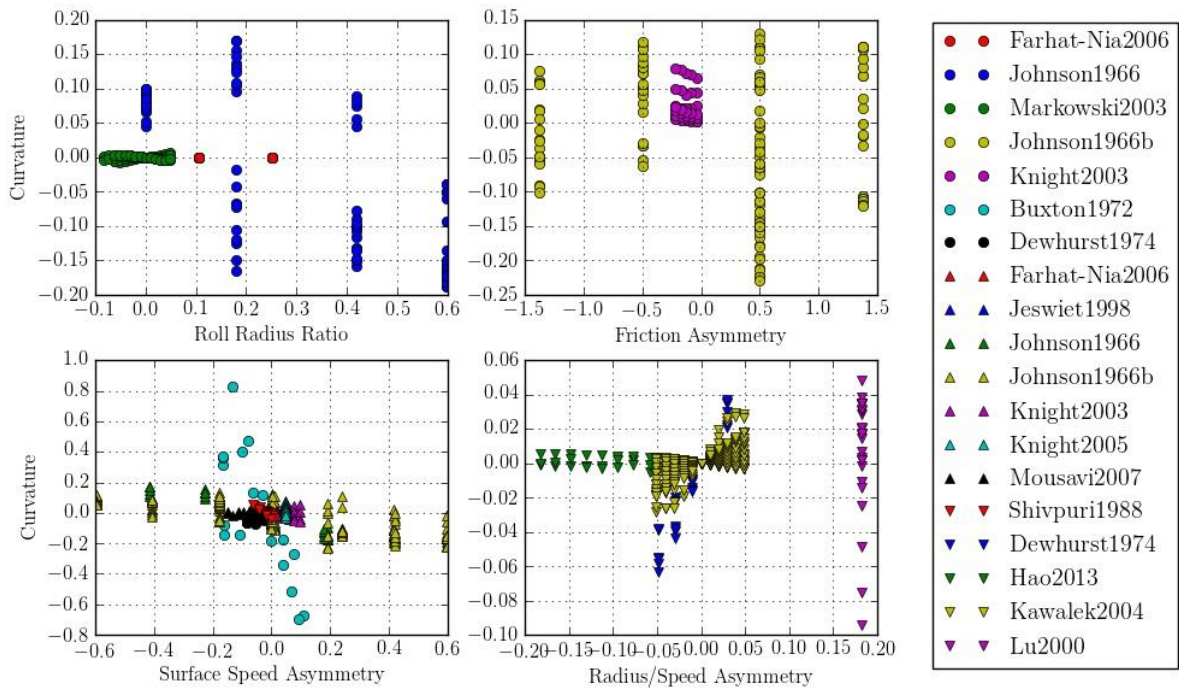


Figure 1 Curvature results of 17 studies grouped by which asymmetry was varied within the study.

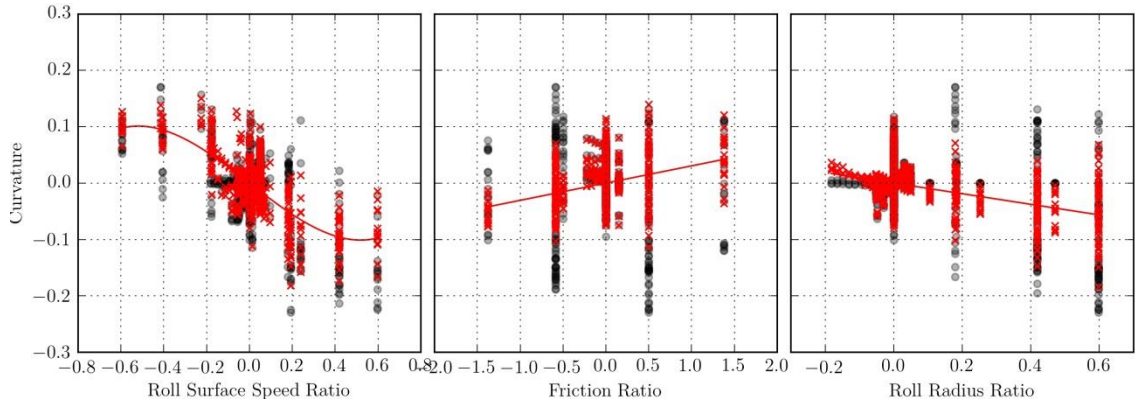


Figure 2 Predictions of the regression model (red line), the errors of the regression model (red crosses) and the original data (black circles) for a roll set-up defined by a roll gap aspect ratio of 0.3, workpiece reduction of 0.2 and yield stress to Young's modulus ratio of 0.00062. Parameters not varied are taken as symmetric.

### 3. Linear Regression

After consideration, the Buxton and Browning [8] data were omitted here as outliers, since the material properties of plasticine are orders of magnitude away from metals. A linear regression is performed on the remaining data.

Multiplying odd powers of the three logged ratios with the other parameters was used to ensure the model remained anti-symmetric (i.e. swapping the properties of the top and bottom rolls should result in the same curvature in the opposite direction). Taking all the resulting cubic order terms produces 47 degrees of freedom, and over-fitting becomes a concern. Hence, only the cubic terms for roll surface speed and roll size were included.

After eliminating terms with a greater than 5% probability of their coefficients equalling zero, the model was reduced to

$$\begin{aligned} \kappa \sim & A_1 \log(\mu_t / \mu_b) + A_2 r \log(\mu_t / \mu_b) + A_3 \log(\delta) \log(\mu_t / \mu_b) + A_4 \log(\sigma_Y / E) \log(\mu_t / \mu_b) \\ & + A_5 \log(U_t / U_b) + A_6 \log(\delta) \log(U_t / U_b) + A_7 \log(\sigma_Y / E) \log(U_t / U_b) \\ & + A_8 \log(U_t / U_b)^3 + A_9 \log(\sigma_Y / E) \log(U_t / U_b)^3 + A_{10} \log(R_t / R_b) + A_{11} r \log(R_t / R_b) \end{aligned}$$

This model with 11 degrees of freedom,  $A_1 \dots A_{11}$ , was fitted over 985 observations, giving an adjusted R-squared value of 0.65. This was as complex as could be reasonably justified given the quantity of data. The regression model's predictions for a particular parameters set are plotted against the three asymmetries in Figure 2. The original data are included for context.

The predictions do not capture sufficient detail to support design or operation decisions. However, they do provide evidence for which dependences exist in the process and which may not. The ratio of yield stress to elastic modulus and the roll gap aspect ratio are seen to be relevant for roll speed asymmetries, reduction for roll size asymmetries and all three for friction asymmetries. A non-linear relationship between the roll surface speed seems likely, however, the cubic radius ratio was found to be insignificant.

It is also estimated from this model that a reduction of 37% renders curvature independent of the roll radius ratio. A similar formula for roll speed independence is found to be

$$\log(\delta) = \sqrt{\frac{17.4 + 2.3 \log(U_t / U_b)}{460.9 + 62.3 \log(\sigma_Y / E)}}$$

This is unlikely to be accurate due the large extrapolation made, however, it provides evidence that an

asymmetric velocity set-up which produces zero curvature would be very sensitive to material properties but should be well behaved and so controllable.

It is emphasized that these conclusions are not claimed to be predictive, but rather should be used to support the design of future studies.

#### 4. Analytic Models

The analytic slab model of Salimi and Sassani [21] was implemented and compared to the data collected here. It was found that only the mean shear component is necessary to reproduce the curvature results within the publication and so that is what is used here. In order to compare results based on friction factor and Coulomb friction, rudimentary slab models are employed to calculate roll torques to determine comparable friction coefficients where necessary.

Figure 3 reproduces Figure 2 with the predictions of the Salimi and Sassani [21] included.

The Salimi and Sassani [21] model captures a key phenomenon as the roll surface speed ratio varies. This can be attributed to the moving neutral points and increasing cross-shear region which slab models are able to predict well. Unfortunately, the larger residual spread can mostly be attributed to material properties which are not featured in this model. It also fails to capture significant variation in either asymmetric friction or roll radius. This is unsurprising due to the simplicity of the model.

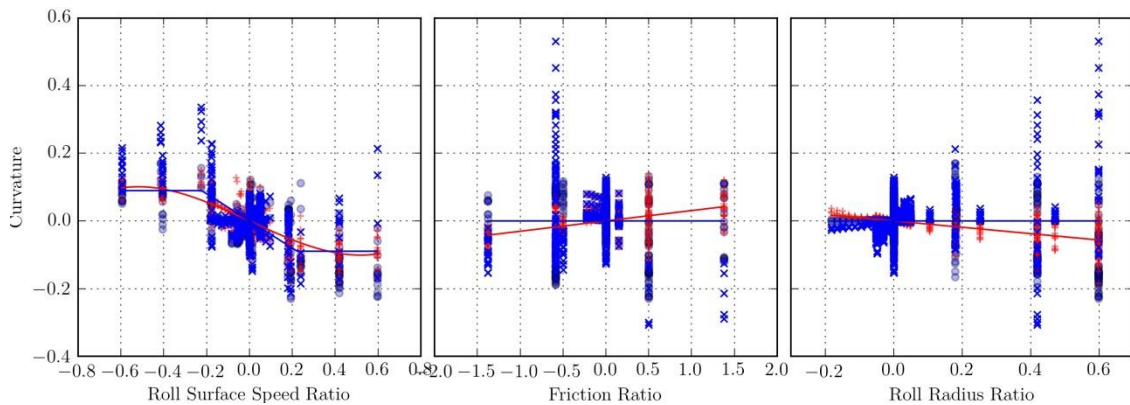


Figure 3 Predictions of the Salimi and Sassani [21] model (solid blue line), the errors of the Salimi and Sassani [21] model (blue crosses), the predictions of the regression model (red dashed line) and associated errors (red pluses) and the original data (black circles) for a roll set-up defined by a roll gap aspect ratio of 0.3, workpiece reduction of 0.2 and yield stress to Young's modulus ratio of 0.00062. Parameters not varied are taken as symmetric.

#### 5. Conclusion

By comparing eight experimental and nine numerical studies from the literature, it is clear that currently no single study properly addresses the complexity of curvature in the asymmetric rolling process. Few studies produce conclusions that apply even moderately generally, due to the complex interactions of the many parameters involved.

It was found that the direction of curvature caused by any of the three asymmetries studied (roll speed ratio, roll radius ratio, and friction ratio) could plausibly be reversed by using different geometric configurations and material properties. The evidence of interdependence of the various parameters also suggests that future studies of the variation in curvature against roll size should be tested with at least several different reductions. Similarly, studies of variations in curvature against asymmetric velocity should be tested with at least several different roll gap aspect ratios and material properties, and studies of variations in curvature against asymmetric friction should be tested with at least several different roll gap aspect ratios, reductions and material properties.

It appears that the mechanisms generating curvature included in analytical, while correct in some cases, are insufficient to capture the full phenomenon. In particular, curvature predictions for asymmetric roll size and friction are lacking. Clearly, curvature resulting from asymmetries in rolling is rather complicated and difficult to predict,

and further experimental, numerical and analytic studies are warranted.

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